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A general aerodynamic theory and a numerical lifting surface technique based on velocity potential formulation for predicting aerodynamic derivatives of a cascade of blades in subsonic flow is described. The unsteady airload prediction method is applied for predicting the flutter boundaries of a single degree-of-freedom in torsion of a cascade of blades. Also, a general flutter program is developed for a two degree-of-freedom staggered cascade in subsonic flow. By utilizing an iterative procedure which permits frequency variation, the flutter frequency and the flutter speed of the reference airfoil are obtained as a function of cascade parameters. The computer program uses very efficient techniques in computing unsteady loads, the flutter frequency, and the flutter speeds.

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AEROELASTIC CHARACTERISTICS OF A CASCADE OF BLADES

TEXAS A & M RESEARCH FOUNDATION
F E BOX H
COLLEGE STATION, TEXAS 77843

February 1978

Final Scientific Report for Period 1 April 74 - 31 December 77

Prepared for

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FOREWORD

This Final Scientific Report was prepared by the Texas A & M Research Foundation under AFOSR Grant 74-2700. Lt. Col. Robert C. Smith, Captain Bob Lawrence, and Lt. Col. G. S. Lewis were the AFOSR Program Managers. William P. Jones and Balusu M. Rao were the Co-principal Investigators during the period April 74 - August 75. Balusu M. Rao was the Principal Investigator from September 75 through December 77.

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INTRODUCTION

The purpose of this report is to give a brief account and summary of the research accomplishments under the present AFOSR Grant to Texas A & M Research Foundation. The unsteady airload prediction technique and an aeroelastic analysis of a cascade of staggered blades in subsonic compressible flow developed under this research grant are discussed. Several research papers and reports have been published from the research conducted and a list of these papers and the summaries are included in the Appendix.

Many researchers have devoted considerable effort towards predicting the flow characteristics through the multiple stages of blades that exist in axial flow compressors. The research efforts have been concentrated on obtaining the aerodynamic loads utilized in the design of more efficient blades. In addition to the need for information about blade aerodynamic loading, flutter characteristics also need to be determined. With the assumption of a two-dimensional flow, the flow through a single row of blades is mathematically equivalent to the flow through a staggered cascade of infinitely many airfoils. Early researchers, Whitehead[1], Kemp and Sears[2], and Schorr and Reddy[3], assumed incompressible flow and developed aerodynamic theories and computational schemes for predicting the unsteady airloads on the blades for oscillatory freestream flow and/or oscillating blades. An example is the work of Kemp and Sears[2] who studied the problem of the unsteady lift generated on a reference airfoil of a cascade. In their approach, they used an oscillatory freestream flow. Their study considered the steady interaction between blades but neglected the unsteady interaction and hence, the effect of cascade spacing. Their approach was to express the unsteady lift as a function of the design parameters, such as the ratio of the airfoil chord and the disturbance wavelength, thus enabling a designer to optimize the performance of a turbomachine design instead of analyzing a particular blade arrangement. In another study, Schorr and Reddy[3] treated the flow through a staggered cascade of airfoils in which the effect of unsteady upstream disturbances were included as in the case of unsteady or distorted inlet flow conditions in an axial flow compressor. Their problem was also formulated under the assumption of incompressible potential flow and numerical results were obtained for oscillatory flow using an approximate solution developed from the integral equations involved. In addition, their solution yielded unsteady lift coefficients for the airfoils as a function of the frequency of the oscillations and for different values of stagger and solidity of the cascade.

In an independent study, Jones and Moore[4] studied the incompressible flow about a cascade of oscillating airfoils at zero mean incidence. In order to obtain a solution, they utilized a unique numerical lifting surface technique which differs from all other methods in that it makes use of the velocity potential instead of acceleration potential doublet distributions. Rao and Jones[5] later applied this technique to the oscillatory flow about an airfoil of a staggered cascade. Airload results obtained for several values of frequency, interblade spacing and stagger angle, showed excellent agreement with the results of Schorr and Reddy[3].

Adopting a technique similar to that of Schorr and Reddy, Fleeter[6] considered the effects of compressibility on both the fluctuating lift and moment coefficients for cascaded airfoils having an upstream non-uniformity. He obtained a solution for the time-dependent, two-dimensional, partial differential equations which describe the perturbation velocity potential through an application of Fourier transform theory. The resulting integral solution equation was evaluated numerically by a matrix inversion technique. The fluctuating lift and moment coefficient variations were computed and represented as a function of Mach number, cascade solidity, cascade stagger angle, interblade phase lag, and reduced frequency. Jones and Moore[7] extended the velocity potential formulation to oscillating two-dimensional airfoils in compressible flow. In their numerical method, they replaced the slowly converging Hankel function series by a rapidly converging exponential series. They studied the effects of varying airfoil spacing, frequency, Mach number, and phase difference between adjacent blades. Variations in the aerodynamic damping can become zero but never negative at certain discrete frequencies. This is a desirable characteristic with respect to flutter due to bending. The results also indicated that the pitching moment aerodynamic damping can become zero relative to the blade quarter-chord axis, while also being zero at the critical frequencies, and could be negative at the higher Mach numbers over a wide range of frequencies of interest in flutter analysis. This is an undesirable characteristic from the standpoint that it increases the area of instability for torsional flutter. Rao and Jones[8] utilized the theory developed in Ref. 7 to determine the airload and moment coefficients on a reference airfoil of a staggered cascade of airfoils in subsonic flow. Circumferential distorsion due to inflow conditions was expressed as an interblade phase lag. Results were obtained for several values of frequency, Mach number, interblade spacing, stagger, and phase lag angles for both cases of oscillatory flows and oscillating blades. The oscillatory flow results compared well with those of Fleeter.

In his study utilizing compressible flow, Whitehead[9] presented calculations for the torsional flutter of a cascade of unstalled blades at zero mean deflection and subsonic Mach numbers. Whitehead found that the effect of increasing Mach number was favorable and tended to suppress the flutter that was predicted by incompressible theory.

In this report the general aerodynamic theory and the numerical lifting surface theory is presented. An investigation is conducted for a single degree of freedom system in torsion. The effect of the flow and geometric parameters is evaluated in establishing flutter boundaries and these results are compared with those of Whitehead[9] who used a completely different computational procedure for calculating the aerodynamic derivatives. Additionally, a general flutter program is applied for a two-degree of freedom (bending-torsion) staggered cascade in subsonic flow. By utilizing an iterative procedure which permits frequency variation, the flutter frequency and the flutter speed of the reference airfoil are obtained as a function of the cascade parameters.

AERODYNAMIC THEORY

General

The governing equation for the unsteady, compressible, two-dimensional flow of an isentropic, inviscid, irrotational fluid is given in terms of its velocity potential by

$$\nabla^2\phi = \frac{1}{a^2}\left[\frac{\partial^2\phi}{\partial t^2} + \frac{\partial}{\partial t}(Q^2) + \vec{Q}\cdot\nabla\left(\frac{Q^2}{2}\right)\right] \tag{1}$$

where ϕ is the perturbation velocity and

$$\vec{Q} = (U+u)\,\vec{i} + w\,\vec{k}\ . \tag{2}$$

U is the freestream velocity. The respective perturbation velocity components along the x a z axes are, $u(=\frac{\partial\phi}{\partial x})$ and $w(=\frac{\partial\phi}{\partial z})$. Assuming that u and w are small compared to U, Eqs. (1) and (2) are combined to yield,

$$(1-M^2)\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial z^2} = \frac{1}{a^2}\frac{\partial^2\phi}{\partial t^2} + \frac{2M}{a}\frac{\partial^2\phi}{\partial x\partial t} \tag{3}$$

where M is the freestream Mach number and a is the speed of sound. The following non-dimensional coordinates and definitions are used:

$$X = \frac{x}{\ell}\ ,\ Z = \beta\frac{z}{\ell}\ ,\ T = \frac{Ut}{\ell} \tag{4}$$

$$\phi(x,z,t) = U\ell\Phi(X,Z)e^{i(\varepsilon X+\omega T)}\ , \tag{5}$$

where

$$\omega = \frac{p\ell}{U}\ ,\ \varepsilon = \frac{M^2\omega}{\beta^2}\ ,\ \text{and}\ \Phi = \Phi(X,Z)\ . \tag{6}$$

When Eq.(3) is combined with Eq.(5), it reduces to a two-dimensional Helmholtz equation for the pertubation velocity in the transformed coordinate system,

$$\nabla^2\Phi + \kappa^2\Phi = 0 \tag{7}$$

where $\kappa = \dfrac{M^2\omega}{\beta^2}$.

For a flow problem, the boundary conditions are usually prescibed. For an isolated airfoil in compressible, unsteady flow, the solution to Eq.(7) can be derived by the application of Green's theorem. A relation for the velocity potential at a given point, Φ_p, is given in terms of velocity potential distribution over the lifting surface and its wake. Treating the lifting surface as a thin airfoil, the discontinuity in the velocity potential between the upper and lower surfaces is expressed as a doublet distribution, $K(= \Phi_{upper} - \Phi_{lower})$. One such solution for an isolated airfoil is given in Ref.7. The relation between the downwash at any point, p, on a thin reference blade on a staggered cascade in subsonic flow and the modified doublet distribution K(X), is given as

$$2\pi W_p = -\int_{-1}^{\infty} K(X)\frac{\partial^2}{\partial Z_p^2} S_o(X_\rho - X, Z_p, D, H, \sigma)dX \tag{8}$$

where,

$$S_o = \frac{\pi i}{2}\sum_{m=-\infty}^{\infty} e^{im(\sigma+\varepsilon D)} H_o^{(2)}\left\{\kappa\left[(X_p - X + mD)^2 + (mH - Z_\rho)^2\right]\right\} \tag{9}$$

and $D = \frac{d}{\ell}$, $H = \frac{\beta h}{\ell}$, and σ is the interblade phase lag. The blades of the cascade are numbered m, with m = 0 as the reference airfoil. Since S_o in Eq. (9) satisfies the wave equation in the form,

$$\frac{\partial^2 S_o}{\partial x^2} + \frac{\partial^2 S_o}{\partial z^2} + \kappa^2 S_o = 0 \tag{10}$$

and in the limit as $z_p \to 0$, Eq. (10) may be rewritten in the form,

$$2\pi Wp = \int_{-1}^{\infty} K(X) \left[\frac{\partial S_1}{\partial X} + \kappa^2 S_o \right] dX \tag{11}$$

where,

$$S_1 = \frac{\partial S_o}{\partial X} = \frac{\pi i \kappa}{2} \sum_{m=-\infty}^{\infty} e^{im(\sigma+\varepsilon D)} \frac{(X_p - X + mD)}{\left[(X_p - X + mD)^2 + m^2 H^2\right]^{\frac{1}{2}}} H_o^{(2)} \left\{ \kappa \left[(X_p - X + mD)^2 + (mH - D)^2 \right] \right\}. \tag{12}$$

The series involving Hankel functions (S_o, S_1) in Eqs.(9) and (12) have very poor convergence characteristics. Therefore, these are replaced by an exponential series as shown in Refs. 10 and 11. However, it is important to understand that this transformation from Hankel function series to exponential series is valid only for an infinite cascade. Hence, it cannot be applied to a finite cascade. The convergence of the exponential series is so good, that the required computational time is less for a cascade when compared to a two-dimensional isolated airfoil in subsonic flow where it is required to use Hankel functions. The transformed relations as given in Ref.7 are,

$$S_o = -\frac{1}{2} \sum_{m=-\infty}^{\infty} \frac{e^{-2\pi a(m) |X_p - X|/S}}{\left[(\delta - m)^2 - \mu^2\right]^{\frac{1}{2}}} \tag{13}$$

and,

$$S_1 = \pm \sum \frac{a(m) e^{-2\pi a(m) |X_p - X|/S}}{\left[(\delta - m)^2 - \mu^2\right]^{\frac{1}{2}}}, \text{ for } X_p > \text{or} < X, \tag{14}$$

where,

$$a(m) = [(\delta - m)^2 - \mu^2]^{\frac{1}{2}} \frac{H}{S} + i(\delta - m)\frac{D}{S}, \text{ for } X_p > \text{or} < X, \tag{15}$$

and,

$$\delta = \frac{\sigma+\varepsilon D}{2\pi}, \quad \mu = \frac{\kappa S}{2\pi}, \text{ and } S = (D^2+H^2)^{\frac{1}{2}} . \tag{16}$$

The exponential form of the series S_o and S_1 not only converge rapidly, but also provide directly the values of critical frequencies as shown in Ref.7. Eqs.(13) and (14) will diverge whenever one of the denominators vanishes and no solution to Eq.(11) would be possible. The critical values of the parameter μ for which the analysis fails is given by,

$$\mu = \delta,\ 1 \pm \delta,\ 2 \pm \delta, \text{ etc.}$$

This phenomenon corresponds to a resonance condition at critical frequencies which constitutes an infinite set of values of a parameter depending on flow and configuration characteristics at which the aerodynamic function becomes infinite everywhere. Resonance conditions are functions of Mach number, frequency, interblade spacing, stagger angle, and acoustic velocity. They represent the condition at which self-induced aerodynamic forces are zero and the blades act effectively as if they were in a vacuum.

Boundary Conditions

The downwash $w(= w'e^{i\omega T})$ can be expressed in terms of (X,Z) by using Eq.(5),

$$w = \frac{\partial\phi}{\partial Z} = \beta U e^{i(\varepsilon X+\bar{\omega}T)} \frac{\partial\Phi}{\partial Z} . \tag{17}$$

Downwash can now be non-dimensionalized to give,

$$W' = \frac{\partial\Phi}{\partial z} = \frac{w'e^{-i\varepsilon x}}{\beta U} . \tag{18}$$

For the oscillating blades with flapping and pitching motions about the mid-chord position, the downwash boundary condition is defined as,

$$w'_i = U[i\omega z' + (1+i\omega x_i)\alpha'] \tag{19}$$

where z' and α' are the amplitudes in flapping and pitching respectively. Since periodic motions were assumed, z and α are defined as

$$z = z'e^{ipt} = z'e^{i\omega T} \tag{20}$$

$$\alpha = \alpha'e^{ipt} = \alpha'e^{i\omega T} . \tag{21}$$

The doublet distributions, K_n's, are complex and depend on z' and α' for given values of ω and M. A typical K_n will be of the form

$$K_n = a_n z' + b_n \alpha' , \tag{22}$$

where a_n and b_n are complex quantities and depend on frequency, Mach number, interblade spacing, stagger, and phase lag.

Numerical Procedure

The numerical procedure developed in Ref. 7 is utlized. The wake boundary condition is given by,

$$K(X) = K_{te}\, e^{-i\nu(X-1)} , \tag{23}$$

where K_{te} is the doublet strength at the trailing edge (X=1). When Eqs. (11) and (23) are combined, they can be expressed in the form,

$$2\pi W_p = \int_{-1}^{1} K(X)\left[\frac{\partial S_1}{\partial X} + \kappa^2 S_o\right] dX + K_{te} I \tag{24}$$

where,

$$I = \int_{1}^{\infty} e^{-i\nu(X-1)}\, K_{te}\left[\frac{\partial S_1}{\partial X} + \kappa^2 S_o\right] dX$$

$$= -S_{1t} - i\nu S_{ot} - \nu^2(1-M^2)P \tag{25}$$

$$P = -\frac{1}{2}\sum_{m=\infty}^{\infty} \frac{Se^{-2\pi a(m)(1-X_p)/S}}{[2\pi a(m)+i\nu s][(\delta-m)^2-\mu^2]^{\frac{1}{2}}} \tag{26}$$

and $a(m) = \left[(\delta-m)^2 - \mu^2\right]^{1/2} \frac{H}{S} - i(\delta-m)\frac{D}{S}$. The symbols S_{ot} and S_{1t} represent S_o and S_1 respectively at the trailing edge.

The airfoil is divided into N equal strips and K is assumed to be K_n, a constant over the nth strip, where n = 1,2,3...,N. Let 2B be the width of each strip and X_n denote the center of the nth strip. In Eq.(24), X_p and W_p are replaced by X_i and W_i respectively, where i refers to the ith strip. Hence Eq.(24) will then be given by,

$$2\pi W_i = \sum_{n=1}^{N} K_n \left[S_1(X_i-X_n-B) - S_1(X_i-X_n+B) + 2\kappa^2 BS_o(X_i-X_n)\right]$$

$$+ K_{te}\, I(X_i), \text{ where } i = 1,2,..,N. \tag{27}$$

When $X_i = X_n$, S_o is evaluated by integrating the exponential form of S_o over the interval $X_n - B \leq X \leq X_n + B$. For other intervals, $X_i \neq X_n$ and S_o can either be found by integration or be taken as the mean value of S_o over the interval. K_{te} can be expressed as a function of K_N using the wake condition. This relation is given by,

$$K_{te} = \frac{K_n}{(2i\nu B + e^{-i\nu B})} . \tag{28}$$

For a given geometry and flow conditions, the numerical terms in Eq.(27) can be evaluated by using Eqs.(13), (14), (25), and (26). For a known set of W_i's, Eq.(27) reduces to a system of N equations with N unknowns, K_1, K_2, ...,K_N, when it is combined with Eq.(28). It is assumed that W_i is known over the airfoil. In Eq.(27), this represents a set of linear algebraic equations given by,

$$2\pi\{W_i\} = [A]\{K\} . \tag{29}$$

Therefore, knowing the values of $\{W_i\}$ and $[A]$ allows for the determination of the doublet distribution, $\{K\}$.

Aerodynamic Derivatives

Euler's equation of motion in unsteady flow is given as,

$$\frac{\partial u}{\partial t} + U\frac{\partial u}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x} . \tag{30}$$

In terms of the upper and lower velocity potential and pressure on the airfoil, Eq. (30) can be shown to be,

$$\frac{\partial k}{\partial t} + U\frac{\partial k}{\partial x} = -\frac{1}{\rho}(p_\ell - p_u) \tag{31}$$

where $k(x) = (\phi_u - \phi_\ell)$. Since the lift per unit chord per unit span is $\tilde{\ell}(x) = (p_\ell - p_u)$, Eq.(31) becomes,

$$\rho\left(\frac{\partial k}{\partial t} + U\frac{\partial k}{\partial x}\right) = \tilde{\ell}(x) . \tag{32}$$

Now from Eq.(5),

$$k(x) = \phi_u - \phi_\ell = U\ell(\Phi_u - \Phi_\ell)e^{i(\varepsilon X + \omega T)} \tag{33a}$$

$$k(X) = U\ell K(X)e^{i(\varepsilon X + \omega T)} \tag{33b}$$

where $K(X) = (\Phi_u - \Phi_\rho)$. Equations (32) and (33) are combined to yield,

$$\tilde{\ell}(X) = \rho U^2\left[i\nu K(X) + \frac{\partial K(X)}{\partial X}\right]e^{i(\varepsilon X + \omega T)} \tag{34}$$

where $\nu = \omega/\beta^2$ Eq.(34) is valid on the airfoil surface, $-1 \leq X \leq 1$. In the wake region, no pressure discontinuities are allowed to exist and hence,

$$i\nu K(X) + \frac{\partial K(X)}{\partial X} = 0 \tag{35}$$

must be satisfied when $X = 1$.

When values of K_n have been obtained, the local lift, $\tilde{\ell}(x)$, at a point X is given by Eq.(34). If $\bar{K}$ is substituted for $Ke^{i\varepsilon X}$, then

$$\frac{\tilde{\ell}(X)}{\rho U^2 \ell} = \left(i\omega\bar{K} + \frac{\partial\bar{K}}{\partial X}\right)e^{i\omega T} \tag{36a}$$

$$\frac{\tilde{\ell}(X)}{\rho U^2 \ell} = \left(i\omega\bar{K} + \frac{\partial\bar{K}}{\partial X}\right)e^{ipT} . \tag{36b}$$

The lift $L(= L'e^{ipt})$ and the nose-up pitching moment about the mid-chord axis $M(= M'e^{ipt})$ are given by,

$$\frac{L'}{\rho U^2 \ell} = \bar{K}_{te} + i\omega \int_{-1}^{1} \bar{K} dX = C_{\ell z} z' + C_{\ell \alpha} \alpha' \tag{37}$$

and,

$$\frac{M'}{\rho U^2 \ell^2} = \bar{K}_{te} + \int_{-1}^{1} \bar{K} dX - i\omega \int_{-1}^{1} X\bar{K} dX = C_{mz} z' + C_{m\alpha} \alpha' \tag{38}$$

where $C_{\ell z}$, $C_{\ell \alpha}$, C_{mz}, and $C_{m\alpha}$ are the aerodynamic derivatives. These are usually complex and depend on the geometry and flow characteristics.

EQUATIONS OF MOTION

The equations of motion for a two degree-of-freedom (bending torsion) system of an airfoil are

$$M\ell \ddot{z} + S\ddot{\alpha} + K_z \ell z = -\rho U^2 \ell (C_{\ell z} z + C_{\ell \alpha} \alpha) \tag{39}$$

$$S\ell \ddot{z} + I\ddot{\alpha} + K_\alpha \alpha = \rho U^2 \ell^2 (C_{mz} z + C_{m\alpha} \alpha) \tag{40}$$

where,

$M = \int dm$ = blade mass per unit span,

$S = \int r\, dm$ = blade static moment about elastic axis,

$I = \int r^2\, dm$ = blade mass moment of inertia about elastic axis,

ℓ = semi-chord length of the blade,

z = flapping motion,

α = pitching motion,

K_z = bending stiffness of the blade

K_α = torsional stiffness of the blade.

Assuming simple harmonic motion,

$$z = z'e^{ipt} = z'e^{i\omega T} \tag{41a}$$

$$\alpha = \alpha'e^{ipt} = \alpha'e^{i\omega T} \tag{41b}$$

and separating the aerodynamic derivatives into real and imaginary parts,

$$C_{\ell z} = C_{\ell zr} + iC_{\ell zI} \tag{42a}$$

$$C_{\ell\alpha} = C_{\ell\alpha R} + iC_{\ell\alpha I} \tag{42b}$$

$$C_{mz} = C_{mzR} + iC_{mzI} \tag{42c}$$

$$C_{m\alpha} = C_{m\alpha R} + iC_{m\alpha I} \tag{42d}$$

Eqs.(39) and (40) reduce to

$$\begin{gathered} (\ell K_z - M\ell p^2 + \rho U^2 \ell C_{\ell z})z' + (\rho U^2 \ell C_{\ell\alpha} - Sp^2)\alpha' = 0 \\ -(\rho U^2 \ell^2 C_{mz} + S\ell p^2)z' + (K_\alpha - \rho U^2 \rho^2 C_{m\alpha} - Ip^2)\alpha' = 0 \ . \end{gathered} \tag{43}$$

Equations (43) are a set of two linear, homogenous equations in z' and α'. For this system to have a nontrivial solution, the coefficient determinant of of z' and α' must be equal to zero.

$$\begin{vmatrix} (\ell K_z - M\ell p^2 + \rho U^2 \ell C_{\ell z}) & (\rho U^2 \ell C_{\ell\alpha} - Sp^2) \\ -(C_{mz}\rho U^2 \ell^2 + S\ell p^2) & (K_\alpha - \rho U^2 \ell^2 C_{m\alpha} - Ip^2) \end{vmatrix} = 0 \tag{44}$$

This determinant is referred to as the flutter determinant and the solution gives the flutter frequency. However, a direct solution cannot be obtained since the arodynamic derivatives are functions of the reduced frequency. Since the aerodynamic derivatives are complex quantities, the determinant can be expressed into two parts, real and imaginary. After expanding the flutter determinant and substituting appropriate values of Eqs.(42), the real and imaginary parts of the flutter determinant become,

$$(\ell K_z K_\alpha)\bar{p}^2 - [\ell K_z(I + \frac{\rho\ell^4}{\omega^2} Cm\alpha R) + K_\alpha(M\ell - \frac{\rho\ell^3}{\omega^2} C_{\ell zR})]\bar{p}$$

$$+ [(M\ell I - S^2\ell) - \frac{\rho\ell^3}{\omega^2}(IC_{\ell zR} - M\ell^2 C_{m\alpha R} + S\ell C_{mzR} - S\ell C_{\ell\alpha R})$$

$$- \frac{\rho^2\ell^7}{\omega^4}(C_{\ell zR}C_{m\alpha R} - C_{\ell zI}C_{m\alpha I} + C_{\ell\alpha I}C_{mzI} - C_{\ell\alpha R}C_{mzR})] = 0 \qquad (45)$$

$$[K_\alpha C_{\rho zI} - \ell^2 K_z C_{m\alpha I}]\ \bar{p} + [S\ell C_{\ell\alpha I} - IC_{\ell zI} + M\ell^2 C_{m\alpha I}$$

$$- S\ell C_{mzI} + \frac{\rho\ell^4}{\omega^2}(C_{\ell\alpha R}C_{mzI} + C_{\ell\alpha I}C_{mzR} - C_{\ell zR}C_{m\alpha I} - C_{\ell zI}C_{m\alpha R})] = 0$$

where $\bar{p} = \frac{1}{p^2}$. If the real and imaginary parts are set to zero, solutions P1 and P2 can be obtained for the real part and P3 for the imaginary part. For an assumed value of ω, if one of the solutions, P1 or P2, of the real part is equal to the solution P3 of the imaginary part, then this ω corresponds to the flutter frequency. However, the aerodynamic derivatives are functions of the Mach number and the reduced frequency. The flutter problem can only be solved after the aerodynamic derivatives are evaluated. It is necessary to assume a Mach number and a reduced frequency and test whether flutter occurs at these values. If the test results are negative, then it is necessary to iterate on reduced frequency until a flutter case occurs at these values. If the two speeds are not equal, then it is necessary to iterate on Mach number until the flutter Mach number is equal to the assumed Mach number.

RESULTS AND DISCUSSION

A single degree-of-freedom analysis for torsion was performed on cascade of blades using the lifting surface aerodynamic theory discussed in a previous section. After obtaining the results, a comparison was made to the results obtained by Whitehead[9] who employed a technique developed by Smith[12] to obtain aerodynamic forces and moments. Whitehead distinguished a region of flutter that he termed as "sub-critical flutter", which occurs in a regime where any acoustic waves generated cannot propagate upstream and downstream. Figure 1 shows the results that Whitehead obtained for a space-to-chord ratio of 1.0, a stagger angle of 45°, a position of the torsional axis at 58.8% chord, and an interblade phase angle of 60°. The imaginary part of the moment coefficient is plotted against the reduced frequency parameter, ω. Also, shown in Fig.1, is a plot of the imaginary part of the moment coefficient from the present calculations. As can be seen, the results obtained by the Rao/Jones lifting surface technique agree closely with the reults obtained by Whitehead. The imaginary part of the moment coefficient is capable of adding energy to the system when it is negative. Assuming that there is no mechanical damping involved, flutter will occur as a net result of energy being added to the system. However, in reporting his results, Whitehead found that he had significant mechanical damping and had to allow for an average value as a flutter limit. This limit is shown in Fig.1. If the value of the imaginary part of the moment coefficient is below this line, the flutter is predicted. The point where it is just possible for the flutter to exist is referred to as a flutter boundary. Whitehead observed that as the Mach number is increased, the points at which flutter is just possible move to progressively lower values of the frequency parameter. This corresponds to higher fluid velocities or to lower blade stiffnesses. The effect of Mach number is therefore highly significant.

Figure 2 shows the frequency parameter below which torsional flutter is just possible and has been plotted against blade axis position. Once again, a comparison is made by utilizing the Rao/Jones technique and Whitehead/Smith technique. The results are in close agreement and concur with Whitehead's observation that increasing the Mach

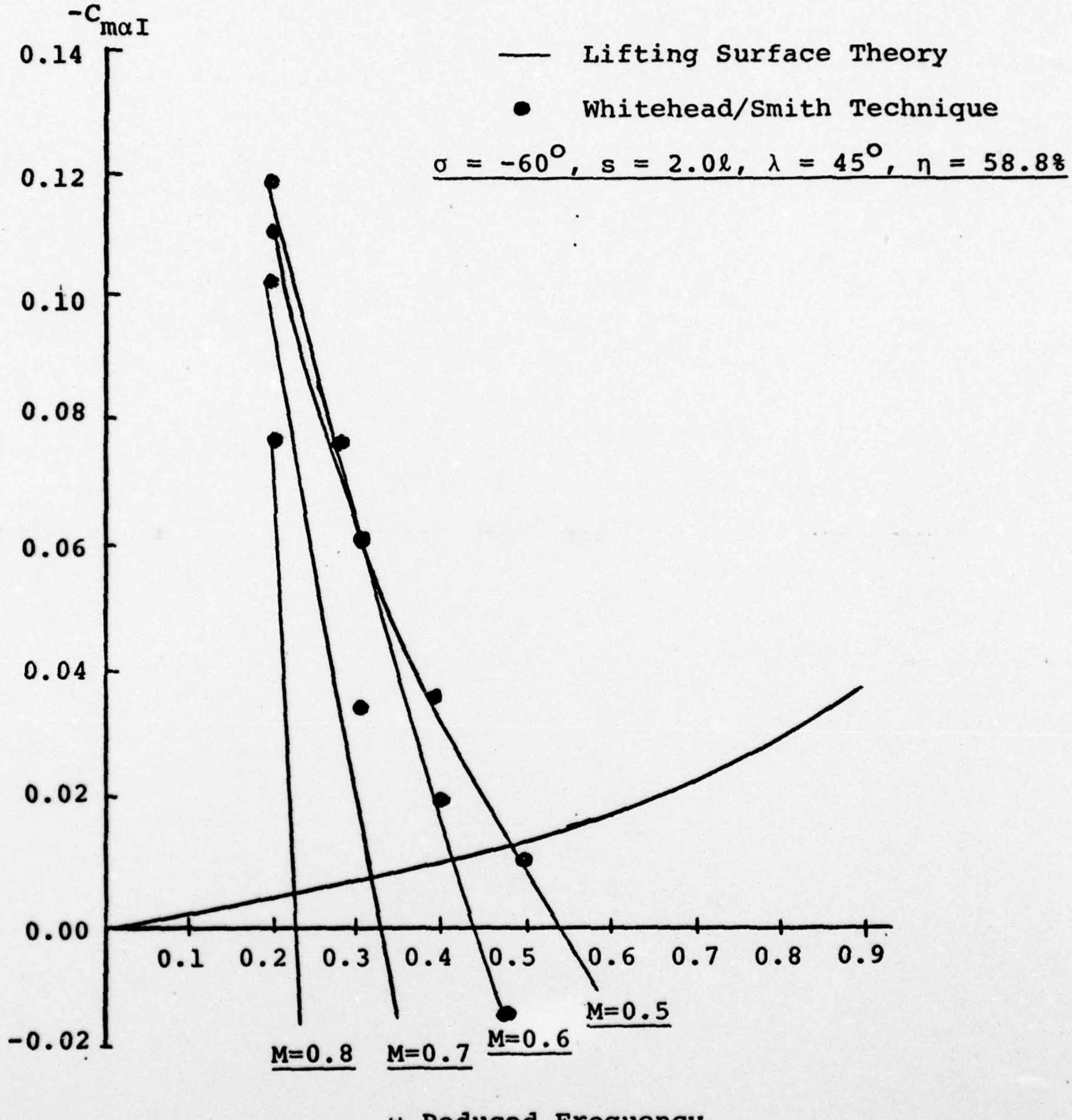

Figure 1. Torsional Flutter Boundaries

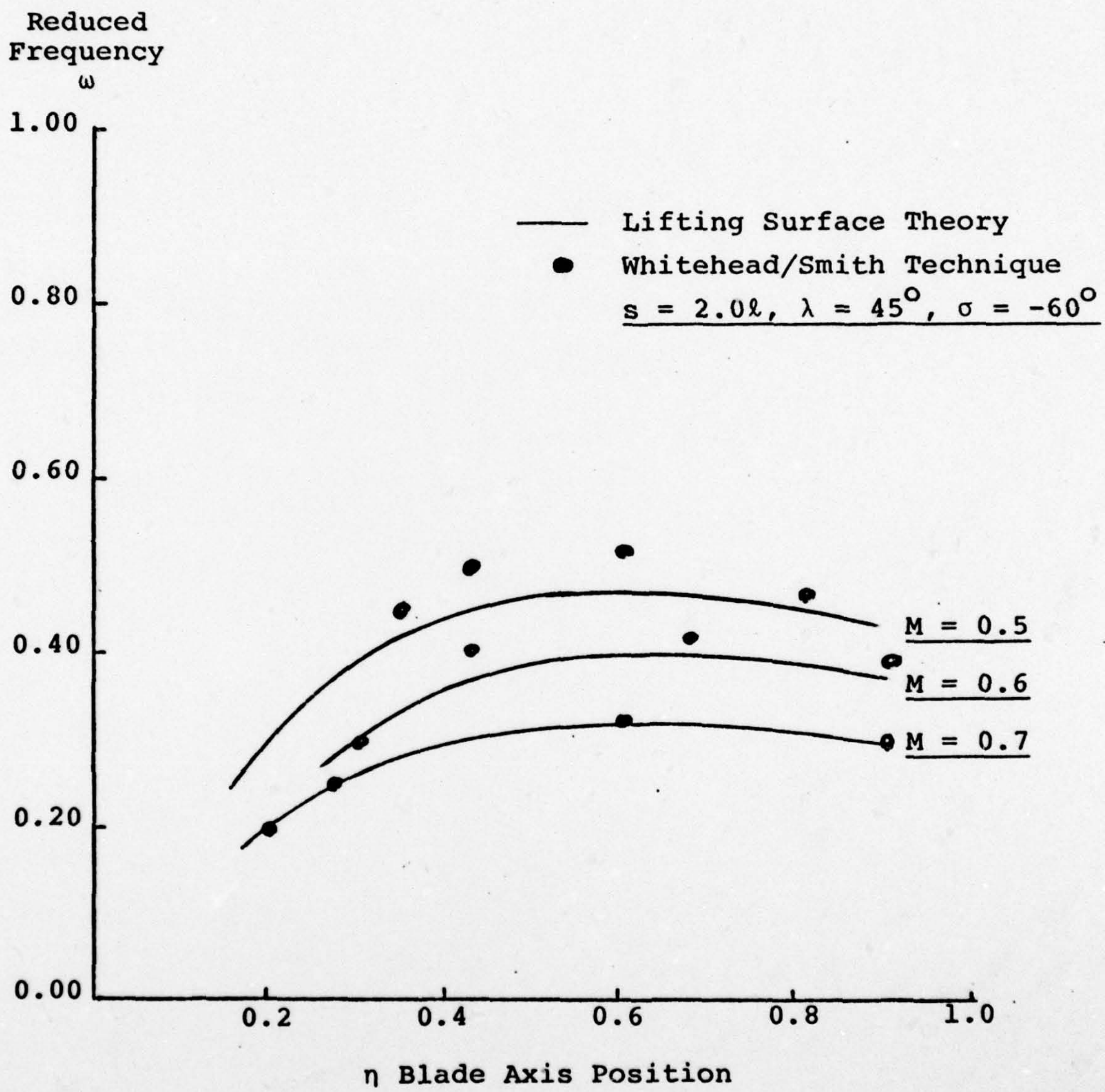

Figure 2. Sub-Critical Flutter

number is favorable and that the worst position for the torsional axis is around fifty to seventy percent chord.

A numerical investigation for two degrees-of-freedom was conducted using the unsteady airlods program interfaced with the flutter analysis program. The geometric and structural properties of the cascade reference airfoil used in this analysis were calculated from the data obtained from Refs.13, 14, and 15. The properties used are:

Static moment about the elastic axis	$S = 5.35\text{x}10^{-4}$ slug-ft
Moment of inertia about the elastic axis	$I = 6.316\text{x}10^{-5}$ slug-ft^2
Mass per unit span	$M = 0.0214$ slugs
Bending natural frequency	$\omega_z = 1000$ rad/sec
Torsional natural frequency	$\omega_\alpha = 2000$ rad/sec
Blade semi-chord length	$\ell = 0.0833$ ft

Figure 3 represents the results obtained for the flutter analysis for various values of interblade spacing ($s = 1.0\ell$, 1.1ℓ, 1.2ℓ, and 1.3ℓ) while the interblade stagger angle was varied from 44° to 60°. Over the range of values considered, it can be seen from Fig.3 that as the interblade stagger angle is increased, the flutter Mach number also increases. This indicates that an increase in the interblade angle would be beneficial in preventing flutter at the lower Mach numbers. This trend is in general agreement with the results reported in Ref. 16.

Figure 4 shows the effect of varying the interblade spacing for various interblade stagger angles ($\lambda = 46^\circ$, 52°, 56°, and 60°). The trend of the curves shows that as the interblade spacing increases for a given interblade stagger angle, the flutter Mach number decreases. This indicates that increasing the interblade spacing will have a destabilizing effect on the cascade of blades. This same trend was also observed in Ref. 16.

A summary of the results of the two degree-of-freedom flutter analysis is presented in the Table.

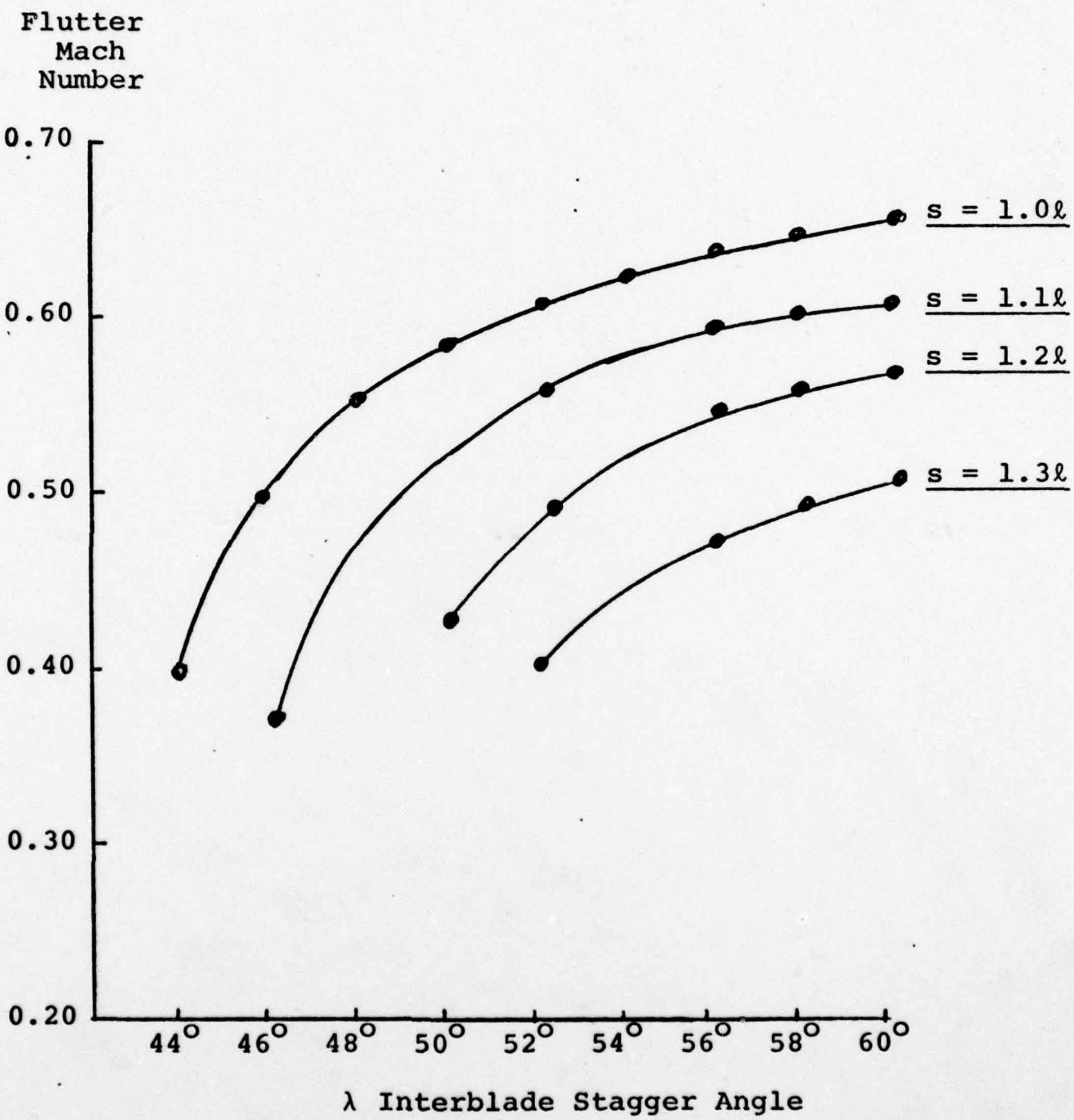

Figure 3. Effect of Interblade Stagger Angle on Flutter Speed

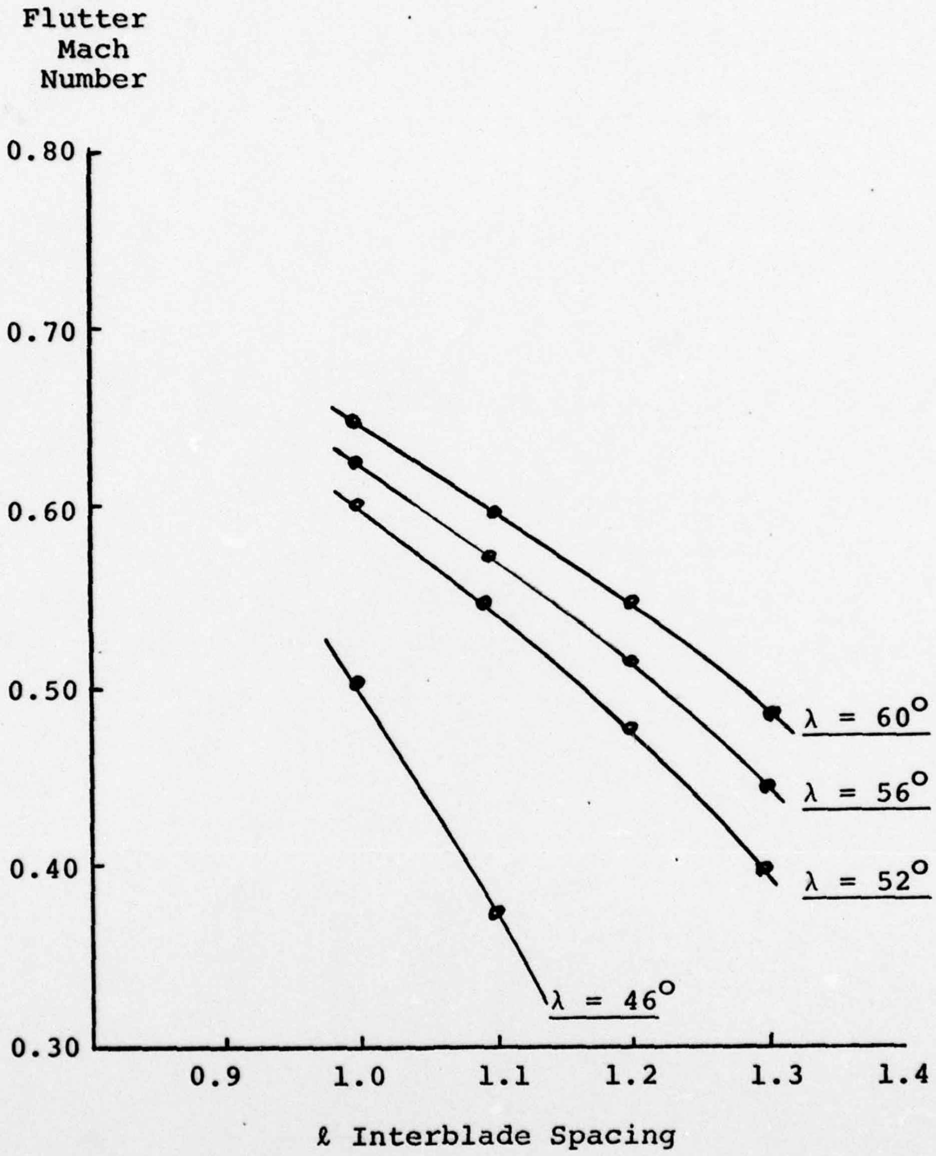

Figure 4. Effect of Interblade Spacing on Flutter Speed

TABLE Summary of Two Degree-of-Freedom Flutter Analysis

Interblade Stagger Angle λ (Degrees)	Interblade Spacing s	Flutter Mach Number M	Flutter Reduced Frequency ω	Interblade Phase Lag σ (Degrees)
44	1.0ℓ	0.4087	0.4539	-30
46	1.0ℓ	0.5028	0.3671	-30
48	1.0ℓ	0.5497	0.3347	-30
50	1.0ℓ	0.5676	0.3234	-30
54	1.0ℓ	0.6109	0.2988	-30
55	1.0ℓ	0.6216	0.2935	-30
56	1.0ℓ	0.6300	0.2888	-30
60	1.0ℓ	0.6445	0.2809	-30
46	1.1ℓ	0.3783	0.4907	-30
52	1.1ℓ	0.5561	0.3301	-30
56	1.1ℓ	0.5843	0.3128	-30
58	1.1ℓ	0.5818	0.3138	-30
60	1.1ℓ	0.6058	0.3001	-30
50	1.2ℓ	0.4300	0.4305	-30
52	1.2ℓ	0.4809	0.3837	-30
56	1.2ℓ	0.5336	0.3440	-30
58	1.2ℓ	0.5406	0.3390	-30
60	1.2ℓ	0.5501	0.3325	-30
52	1.3ℓ	0.4045	0.4580	-30
56	1.3ℓ	0.4715	0.3911	-30
58	1.3ℓ	0.4757	0.3872	-30
60	1.3ℓ	0.4990	0.3682	-30

CONCLUSIONS AND RECOMMENDATIONS

A unique velocity potential lifting surface technique has been developed and used to determine the unsteady airloads for an infinite cascade of staggered blades in subsonic flow. The unsteady airloads were then utilized in a FORTRAN program to perform a flutter analysis for a single degree-of-freedom system in torsion and for a two degree-of-freedom system in bending and torsion.

Several geometric and flow parameters of a staggered cascade were varied over a specific range of values and were found to influence the cascade in the following manner:

1. If Mach number is increased, it will have an increased damping effect on the stability of the cascade in pitching motion.
2. If interblade spacing is increased, the damping decreases while still below the first critical frequency is equal to zero at the first critical frequency.
3. As interblade stagger angle increases, the lowest critical frequency decreases.
4. As interblade phase lag is increased with increasing reduced frequency, the damping will decrease rapidly to zero for an interblade phase lag of 180 degrees.

When the cascade was analyzed for a single degree-of-freedom system in torsion, results were found to compare favorably with the results obtained by Whitehead for flutter boundaries. A comparison of the lifting surface technique employed by Rao and Jones and a different technique utilized by Whitehead in determining torsional flutter, were shown to be in agreement in indicating that increasing Mach number is a desirable condition from the standpoint of flutter.

The developed computer program is general and is capable of predicting flutter analysis of a two degree-of-freedom system of an infinite cascade in subsonic flow. It was concluded that increasing interblade stagger angle would be beneficial in preventing flutter at low Mach numbers while an increase in interblade spacing would produce a destabilizing influence on the cascade of blades.

While a new flutter program was developed to determine the cascade flutter speed and frequency, only a limited range of values were analyzed. The program uses very efficient techniques in computing unsteady airloads, the flutter frequency, and the flutter speeds. Further investigation should be conducted over a large range of cascade parameter values for different geometric configurations, before one can attempt to make general conclusions.

APPENDIX - PUBLISHED RESEARCH PAPERS AND REPORTS

1. Kronenberger, Jr. L., "Flutter Analysis of a Cascade of Staggered Blades in Subsonic Flow," M.S. Thesis, Texas A&M University, December 1977.

   The purpose of the report is to utilize a numerical lifting surface theory developed by Rao and Jones to predict the unsteady airloads on a cascade of staggered blades in subsonic compressible flow. An investigation is conducted to determine the effect on the unsteady airloads when parameters such as reduced frequency, interblade staggered angle interblade spacing, and interblade spacing, and interblade phase lag are varied over a specific range of values. Once the unsteady airloads have been determined, they are used to perform an aeroelastic analysis of the staggered cascade for a single degree of freedom in torsion and a two degree-of-freedom system in bending and torsion.

   Results of the single degree of freedom analysis yield flutter boundaries. These are compared to results obtained by Whitehead who used a different technique for calculating the unsteady airloads on a finite cascade. A new general flutter program is developed for the two degree-freedom-system. The airloads are used as forcing functions in the resulting two *Lagrangean equations* of motion representing the bending and torsional degrees of freedom. The iterative procedure of the flutter program yields the flutter frequency and speed of the cascade reference airfoil as a function of the cascade parameters.

2. White, G.P., "Flutter Analysis of a Cascade of Rotor Blades," Presented at the AIAA 13th Annual Meeting and Technical Display, January 1977; also presented at the 24th Annual AIAA South-West Student Paper Competion (won the First Place Award in the Undergraduate Division), April 1976.

   A classical two-dimensional, bending-torsion flutter analysis of a reference airfoil in a cascade of infinite blades is performed. The unsteady airloads on the reference airfoil are predicted using a numerical lifting surface theory. Several cascade and flow parameters such as interblade spacing, stagger, phase angle between blades, Mach number, and frequency are investigated. The bending-torsion flutter speed of the cascaded reference airfoil is studied as a function of the cascade and flow parameters and the results are compared with that of an isolated airfoil.

3. Jones, W.P. and Moore, J.A., "Aerodynamic Theory for a Cascade of Oscillating Airfoils in Subsonic Flow," AIAA Journal, Vol. 14, No. 5, May 1976, pp. 601-605.

   The theory is developed in terms of the velocity potential rather than the acceleration potential, and a brief outline is

given of the simple numerical technique used. The effects of varying airfoil spacing, frequency, Mach number, stagger angle, and phase difference between adjacent airfoils are discussed. Particular attention is given to variations in the aerodynamic damping for pure vertical translational and pitching motions. It is shown that the translational damping can become zero at certain discrete frequencies but that it never becomes negative. The pitching moment damping, however, can become negative over a wide range of frequenicies of practical interest. The airfoils are assumed to be zero mean incidence.

4. Rao, B.M. and Jones, W.P., "Unsteady Airloads on a Cascade of Staggered Blades in Subsonic Flow," Presented at the 46th Meeting of the AGARD Propulsion and Energetics Panel, Unsteady Phenomena in Turbomachinery, September 1975. AGARD-CP-177.

   The Jones-Moore numerical lifting surface technique is applied to the theory developed by Jones to predict the airloads and moments on an airfoil of a staggered cascade of rotor blades in subsonic flow. Circumferential distortion due to inlet flow conditions is expressed as an interblade phase lag and both cases of oscillating airfoils and oscillatory inflow were considered. Results obtained for several values of frequency, stagger angle, blade spacing, and interblade phase lag.

5. Rao, B.M. and Jones, W.P., "Unsteady Airloads on a Cascade of Staggered Blades in Incompressible Flow," Proceeding of a Workshop on Unsteady Flows in Jet Engines, Sponsored by the Project Squid (US Navy), AFOSR, and UARL, November 1974.

   The Jones-Moore numerical lifting surface technique is applied to predict the airloads in oscillatory flow on an airfoil of a staggered cascade in incompressible flow. Circumferential distortion due to inlet flow conditions is expressed as a phase lag between blades as suggested by Schorr and Reddy. Also, the results are obtained for one combination of stagger angle and blade spacing at several phase lag angles and frequencies.

REFERENCES

1. Whitehead, D.S., "Bending Flutter of Unstalled Cascade of Blades at Finite Deflection," British Aeronautical Research Council, R&M 3386, October 1962.

2. Kemp, N.H. and Sears, W.R., "The Unsteady Forces due to Viscous Wakes in Turbomachines," Journal of the Aeronautical Sciences, Vol. 22, 1955, pp. 478-483.

3. Schorr, B. and Reddy, K.C., "Inviscid Flow Through Cascades in Oscillatory and Distorted Flow," AIAA Journal, Vol. 9, No. 11, October 1971, pp. 2043-2050.

4. Jones, W.P. and Moore, J.A., "Flow in the Wake of a Cascade of Oscillating Airfoils," AIAA Journal, Vol. 10, No. 12, December 1972, pp. 1600-1605.

5. Rao, B.M. and Jones, W.P., "Unsteady Airloads on a Cascade of Staggered Blades in Incompressible Flow," Presented at the Workshop on Unsteady Flow in Jet Engines, United Aircraft Research Laboratories, July 1974.

6. Fleeter, S., "Fluctuating Lift and Moment Coefficients for Cascaded Airfoils in a Nonuniform Compressible Flow," Journal of Aircraft, Vol. 10, No. 2, February 1973, pp. 93-98.

7. Jones, W.P. and Moore, J.A., "Aerodynamic Theory for a Cascade of Oscillating Airfoils in Compressible Subsonic Flow," AIAA Journal, Vol. 14, No. 5, May 1976., pp. 601-605.

8. Rao, B.M. and Jones, W.P., "Unsteady Airloads on a Cascade of Staggered Blades in Subsonic Flow," Presented at the 46th Propulsion and Energetics Panel Meeting, Advisory Group for Aerospace Research and Development, September 1975.

9. Whitehead, D.S., "The Effect of Compressibility on Unstalled Torsional Flutter," British Aeronautical Research Council, R&M 3754, August 1973.

10. Infeld, L., Smith, V.G., and Chien, W.Z., "On Some Series of Bessel Functions," Journal of Mathematics and Physics, Vol. XXVI, No. 1, April 1947, pp. 22-28.

11. Kaji, S. and Okazaki, T., "Propagation of Sound Waves Through a Blade Row," Journal of Sound and Vibration, Vol. 11, No. 3, 1970, Parts I and II, pp. 339-375.

12. Smith, S.N., "Discrete Frequency Sound Generation in Axial Flow Turbomachines," British Aeronautical Research Council, R&M 3709, 1972.

13. Sawyer, J.W., "Gas Turbine Engineering Handbook," Gas Turbine Publications, Inc., Stratford, Connecticut, 1966.

14. Emery, J.C., "Systematic Two-Dimensional Cascade Test of NACA 65-Series Compressor Blades at Low Speeds," NACA TR-1368, U.S. Government Printing Office, 1969.

15. Johnson, I.A. and Bullock, R.O., "Aerodynamic Design of Axial-Flow Compressors," NASA SP-36, 1965.

16. White, G.P., "Flutter Analysis of a Cascade of Rotor Blades," Prsented at the AIAA 13th Annual Meeting and Technical Display, January 1977.